

Given the information in each question below, create a sketch of the function.

Question 1:

Function	$f(x) = -2x^3 + 6x^2 - 3$
Domain	$(-\infty, \infty)$
Intercepts	X: $x = -0.6, x = 0.8, x = 2.8$ Y: $y = -3$
Holes	N/A
Asymptotes	N/A
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = -\infty, \lim_{x \rightarrow -\infty} f(x) = \infty$
Intervals of increasing and decreasing	Decreasing: $(-\infty, 0) \cup (2, \infty)$ Increasing: $(0, 2)$
Intervals of concavity	Concave down: $(1, \infty)$ Concave up: $(-\infty, 1)$
Key Points	Local maxima: $(2, 5)$ Local minima: $(0, -3)$ Inflection values: $(1, 1)$

Question 2:

Function	$f(x) = 4x^3 - x^4$
Domain	$(-\infty, \infty)$
Intercepts	X: $x = 0, x = 4$ Y: $y = 0$
Holes	N/A
Asymptotes	N/A
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = -\infty$
Intervals of increasing and decreasing	Increasing: $(-\infty, 0) \cup (0, 3)$ Decreasing: $(3, \infty)$
Intervals of concavity	Concave up: $(0, 2)$ Concave down: $(-\infty, 0) \cup (2, \infty)$
Key Points	Local maxima: $(3, 27)$ Local minima: N/A Inflection values: $(0, 0), (2, 16)$

Question 3:

Function	$f(x) = 2x - 3x^{\frac{2}{3}}$
Domain	$(-\infty, \infty)$
Intercepts	X: $x = 0, x = 3.4$ Y: $y = 0$
Holes	N/A
Asymptotes	N/A
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = \infty$ $\lim_{x \rightarrow -\infty} f(x) = -\infty$
Intervals of increasing and decreasing	Increasing: $(-\infty, 0) \cup (1, \infty)$ Decreasing: $(0, 1)$
Intervals of concavity	Concave up: $(-\infty, 0) \cup (0, \infty)$ Concave down: N/A
Key points	Local maxima $(0, 0)$ Local minima: $(1, -1)$ Inflection values: N/A

Question 4:

Function	$f(x) = x^{\frac{2}{3}} \left(\frac{5}{2} - x \right)$
Domain	$(-\infty, \infty)$
Intercepts	X: $x = 0, x = 2.5$ Y: $y = 0$
Holes	N/A
Asymptotes	N/A
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = -\infty$ $\lim_{x \rightarrow -\infty} f(x) = \infty$
Intervals of increasing and decreasing	Increasing: $(0, 1)$ Decreasing: $(-\infty, 0) \cup (1, \infty)$
Intervals of concavity	Concave up: $(-\infty, -0.5)$ Concave down: $(-0.5, 0) \cup (0, \infty)$
Key Points	Local maxima: $(1, 1.5)$ Local minima: $(0, 0)$ Inflection values: $(-0.5, 1.9)$

Question 5:

Function	$f(x) = \frac{x^2-3}{x-2}$
Domain	$(-\infty, 2) \cup (2, \infty)$
Intercepts	X: $x = -1.7, x = 1.7$ Y: $y = 1.5$
Holes	N/A
Asymptotes	Vertical: $x = 2$ Horizontal: N/A
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = \infty$ $\lim_{x \rightarrow -\infty} f(x) = -\infty$
Intervals of increasing and decreasing	Increasing: $(-\infty, 1) \cup (3, \infty)$ Decreasing: $(1, 2) \cup (2, 3)$
Intervals of concavity	Concave up: $(2, \infty)$ Concave down: $(-\infty, 2)$
Key Points	Local maxima: $(1, 2)$ Local minima: $(3, 6)$ Inflection values: N/A

Question 6:

Function	$f(x) = \frac{8x}{x^2+4}$
Domain	$(-\infty, \infty)$
Intercepts	X: $x = 0$ Y: $y = 0$
Holes	N/A
Asymptotes	Horizontal Asymptote at $y = 0$
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = 0$ $\lim_{x \rightarrow -\infty} f(x) = 0$
Intervals of increasing and decreasing	Increasing: $(-2, 2)$ Decreasing: $(-\infty, -2) \cup (2, \infty)$
Intervals of concavity	Concave up: $(-3.5, 0) \cup (3.5, \infty)$ Concave down: $(-\infty, -3.5) \cup (0, 3.5)$
Key Points	Local maxima: $(2, 2)$ Local minima: $(-2, -2)$ Inflection values: $(-3.5, -1.7), (0, 0), (3.5, 1.7)$

Question 7:

Function	$f(x) = xe^{\frac{1}{x}}$
Domain	$(-\infty, 0) \cup (0, \infty)$
Intercepts	X: N/A Y: N/A
Holes	(0,0)
Asymptotes	Vertical: $x = 0$ Horizontal: N/A
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = \infty$ $\lim_{x \rightarrow -\infty} f(x) = -\infty$
Intervals of increasing and decreasing	Increasing: $(-\infty, 0) \cup (1, \infty)$ Decreasing: $(0, 1)$
Intervals of concavity	Concave up: $(0, \infty)$ Concave down: $(-\infty, 0)$
Key Points	Local maxima: N/A Local minima: (1, 2.7) Inflection values: N/A

Question 8:

Function	$f(x) = \ln(4 - x^2)$
Domain	$(-2, 2)$
Intercepts	X: $x = -1.7, x = 1.7$ Y: $y = 1.4$
Asymptotes	Vertical: $x = -2, x = 2$ Horizontal: N/A
Holes	N/A
Limits to ∞ and $-\infty$	Not possible as there is a restriction on both sides of the domain.
Intervals of increasing and decreasing	Increasing: $(-2, 0)$ Decreasing: $(0, 2)$
Intervals of concavity	Concave up: N/A Concave down: $(-2, 2)$
Key Points	Local maxima: (0, 1.4) Local minima: N/A Inflection values: N/A

Question 9:

Function	$f(x) = \frac{1}{1+e^{-x}}$
Domain	$(-\infty, \infty)$
Intercepts	X: N/A Y: $y = 1$
Holes	N/A
Asymptotes	Vertical: N/A Horizontal: $y = 0, y = 1$
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = 1$ $\lim_{x \rightarrow -\infty} f(x) = 0$
Intervals of increasing and decreasing	Increasing: $(-\infty, \infty)$ Decreasing: N/A
Intervals of concavity	Concave up: $(-\infty, 0)$ Concave down: $(0, \infty)$
Key Points	Local maxima: N/A Local minima: N/A Inflection values: $x = 0$

Question 10:

Function	$f(x) = \frac{2x^2+x-1}{x^2-1}$
Domain	$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
Intercepts	X: $x = 0.5$ Y: $y = 1$
Holes	$(-1, 1.5)$
Asymptotes	Vertical: $x = 1$ Horizontal: $y = 2$
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 2$
Intervals of increasing and decreasing	Increasing: N/A Decreasing: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
Intervals of concavity	Concave up: $(1, \infty)$ Concave down: $(-\infty, -1) \cup (-1, 1)$
Key Points	Local maxima: N/A Local minima: N/A Inflection values: N/A

Question 11:

Function	$f(x) = \frac{1}{x^2-1}$
Domain	$(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
Intercepts	X: N/A Y: $y = -1$
Holes	N/A
Asymptotes	Vertical: $x = -1, x = 1$ Horizontal: $y = 0$
Limits to ∞ and $-\infty$	$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$
Intervals of increasing and decreasing	Increasing: $(-\infty, -1) \cup (-1, 0)$ Decreasing: $(0, 1) \cup (1, \infty)$
Intervals of concavity	Concave up: $(-\infty, -1) \cup (1, \infty)$ Concave down: $(-1, 1)$
Key Points	Local maxima: $(0, -1)$ Local minima: N/A Inflection values: N/A